Calculus 1 Worksheet: Derivatives

Instructions

Find the derivative of each function with respect to x. Show all your steps.

Problems

1. **Power Rule**
   Find $f'(x)$ given:
   $$f(x) = 4x^5 - 3x^3 + 7x - 2$$

2. **Product Rule**
   Find $f'(x)$ given:
   $$f(x) = x^2 e^x$$

3. **Quotient Rule**
   Find $f'(x)$ given:
   $$f(x) = \frac{3x + 1}{x^2 - 2}$$

4. **Chain Rule (Polynomial)**
   Find $f'(x)$ given:
   $$f(x) = (2x^3 + 5)^4$$

5. **Chain Rule (Trigonometric/Logarithmic)**
   Find $f'(x)$ given:
   $$f(x) = \ln(\cos(x))$$

Solutions

1. **Solution for:** $f(x) = 4x^5 - 3x^3 + 7x - 2$
   Using the Power Rule $\frac{d}{dx}(ax^n) = anx^{n-1}$:

$$f'(x) = \frac{d}{dx}(4x^5) - \frac{d}{dx}(3x^3) + \frac{d}{dx}(7x) - \frac{d}{dx}(2)$$
$$f'(x) = 4(5x^4) - 3(3x^2) + 7(1) - 0$$
$$f'(x) = \mathbf{20x^4 - 9x^2 + 7}$$

2. **Solution for:** $f(x) = x^2e^x$
   Using the Product Rule $(uv)' = u'v + uv'$ where $u = x^2$ and $v = e^x$:

$$u' = 2x$$
$$v' = e^x$$
$$f'(x) = (2x)(e^x) + (x^2)(e^x)$$
$$f'(x) = \mathbf{e^x(2x + x^2)}$$

3. **Solution for:** $f(x) = \frac{3x+1}{x^2-2}$
   Using the Quotient Rule $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $u = 3x + 1$ and $v = x^2 - 2$:

$$u' = 3$$
$$v' = 2x$$
$$f'(x) = \frac{(3)(x^2-2) - (3x+1)(2x)}{(x^2-2)^2}$$
$$f'(x) = \frac{3x^2 - 6 - (6x^2 + 2x)}{(x^2-2)^2}$$
$$f'(x) = \frac{3x^2 - 6 - 6x^2 - 2x}{(x^2-2)^2}$$
$$f'(x) = \mathbf{\frac{-3x^2 - 2x - 6}{(x^2-2)^2}}$$

4. **Solution for:** $f(x) = (2x^3 + 5)^4$
   Using the Chain Rule $\frac{d}{dx}[g(h(x))] = g'(h(x)) \cdot h'(x)$.
   Let $u = 2x^3 + 5$, then $y = u^4$.

$$\frac{dy}{du} = 4u^3$$
$$\frac{du}{dx} = 6x^2$$
$$f'(x) = 4(2x^3 + 5)^3 \cdot (6x^2)$$
$$f'(x) = \mathbf{24x^2(2x^3 + 5)^3}$$

5. Solution for: $f(x) = \ln(\cos(x))$

Using the Chain Rule. Recall that $\frac{d}{dx}(\ln(u)) = \frac{1}{u} \cdot u'$.

$$\begin{aligned}
u &= \cos(x) \\
u' &= -\sin(x) \\
f'(x) &= \frac{1}{\cos(x)} \cdot (-\sin(x)) \\
f'(x) &= -\frac{\sin(x)}{\cos(x)} \\
f'(x) &= -\tan(\mathbf{x})
\end{aligned}$$